

## Problems undergoing the test in Mathematical Statistics

(2014-06-11, Corporate Management):

1. Give formulae for sums  $1+2+3+\dots+n$ ,  $1^2+2^2+3^2+\dots+n^2$  ( $n \in \mathbf{N}$ ) and prove them.
2. Define the  $n$ th triangular number,  $T_n$ , state the recursive formula for  $T_n$ , derive the concise formula for  $T_n$ .
3. Formulate the handshake problem, produce the recurrence for  $H_n$  ( $H_n$  stands for the number of handshakes exchanged by  $n$  people), obtain the concise formula for  $H_n$ .
4. Discuss permutations with and without repetitions, as well as their numbers.
5. Discuss variations with and without repetitions as well as their numbers.
6. Formulate and prove (via the mathematical induction) the binomial theorem.
7. Give the definition of the binomial coefficient  $\binom{n}{k}$ , prove the recurrence for it, discuss where it appears.
8. Present the Pascal triangle. Connect it to the Manhattan walking.
9. Discuss the formula for  $(1+x)^r$  when  $r \in \mathbf{R}$ .
10. Discuss the  $n$ th falling power  $x^{\overline{n}}$  and the  $n$ th rising power  $x^{\underline{n}}$ .
11. Present Stirling numbers of the 1st kind  $\left[ \begin{smallmatrix} n \\ k \end{smallmatrix} \right]$ , aka cycle numbers.
12. Present Stirling numbers of the 2nd kind  $\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$ , aka subset numbers.
13. Discuss the Stirling formula (giving an approximation for  $n!$ ).
14. Define the Euler gamma function  $\Gamma$  and discuss its relation to the factorial function.
15. Calculate  $\Gamma(1/2)$ .
16. Give the classical (aka Laplace) definition of the probability, Pr.
17. List properties of the probability (including the Venn diagram illustrations).
18. Define the conditional probability  $\Pr(B|A)$  (saying how we word it).
19. Present the multiplication rule for the probability and the law of total probability.
20. Discuss Bayes' formula.
21. Discuss the Bayesian updating (in terms of hypotheses K and G and evidences P and N; K as 'krank', G as 'gesund', P as positive, N as negative).
22. Explain the pair-matching table (with explanation about K and G, P and N) and its numeric correspondences: cross quantitative matrix, cross frequentative matrix.
23. Define the sensitivity (Sens) and the specificity (Spec), the significance ( $\alpha = 1 - \text{Sens}$ ) and the power ( $\beta = 1 - \text{Spec}$ ) of the test.
24. Discuss the hypothesis testing - use such notions as empirical distribution, null and alternative hypothesis ( $H_0$  and  $H_1$ ) test statistics  $t_{\text{empir}}$  and  $t_{\text{theor}}$ , rejection region  $R_{\text{theor}}$ .
25. Discuss the DiscreteTriangular(2) distribution.
26. Discuss the Bernoulli( $p$ ) and Binomial( $n, p$ ) distributions.
27. Discuss the Poisson( $\lambda$ ) distribution, incl. calculation of  $\sum_{k=0, 1, 2, \dots} f_k$ ,  $E(X)$  and  $\text{Var}(X)$ .
28. Discuss the Gauss( $\mu, \sigma$ ) distribution, aka Normal( $\mu, \sigma$ ) distribution, including calculation of  $\sum_{k=0, 1, 2, \dots} f_k$ ,  $E(X)$  and  $\text{Var}(X)$ .
29. \*Discuss the Exponential( $\lambda$ ) distribution.
30. \*Discuss the ChiSquared( $m$ ) distribution.
31. \*Discuss the Gamma( $k, s$ ) distribution.
32. Define PMF and PDF, CDF and \*MGF of a (discrete, continuous) random variable.
33. \*Define the geometrical probability and present Bertrand paradox.

\* Subjects not realized due to the cancellations of classes (5/15=20%). Students aiming to get the highest mark (A = bardzo dobry) are asked to master these topics.